

Quiz

02/15/2013

The test lasts 25 minutes. No documents are allowed. The use of a calculator, cell phone or other equivalent electronic device is not allowed.

Exercise 1: We want to give an approximate solution $(u_i)_{i \in \{0, \dots, n\}}$ of the differential equation

$$(0.1) \quad \begin{cases} -\frac{d^2 u}{dx^2}(x) = 4\pi^2 \cos(2\pi x), & \forall x \in [0, 1] \\ u(0) = u(1) = 0. \end{cases}$$

We define

$$h = \frac{1}{n+1}$$

and set $x_i = ih$ for $i \in \{0, \dots, n\}$. We will take $n = 3$ (and then $h = 1/4$).

(1) For $i \in \{1, 2, 3\}$, write an approximation of equation (0.1) at x_i .

Tip: we have

$$\frac{d^2 u}{dx^2}(x_i) \simeq \frac{1}{h^2} (u_{i+1} - 2u_i + u_{i-1}).$$

(2) Rewrite the system of approximate equations obtained as a linear system of the form

$$AV = F$$

for $F \in \mathbb{R}^3$,

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \text{ and } V = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

(3) Solve the system with the LU method to find V .

(4) *Bonus question:*¹ Compare V with the exact solution of (0.1), given by

$$u(x) = \cos(2\pi x).$$

¹Will give extra points.